©2017 Published in 5th International Symposium on Innovative Technologies in Engineering and Science 29-30 September 2017 (ISITES2017 Baku - Azerbaijan)

Sound Transmission in an Expansion Chamber with Lined Walls and Extended Inlet/Outlet

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Abstract

The transmission of sound waves in an expansion chamber with extended inlet/outlet is investigated in the case the walls of the expansion chamber and the extended inlet/outlet are lined with different acoustically absorbent materials which are characterized mathematically by impedance boundary conditions. By using series expansions in extended inlet/outlet regions and using Fourier transform technique elsewhere we obtain a Wiener-Hopf equation whose solution involve a set of infinitely many unknown expansion coefficients satisfying a system of linear algebraic equations. Coupling series expansions and the field representations results with a solution involving infinite series. The solution of the algebraic system and the W-H equation is obtained numerically by truncating the infinite series at some number N and then the variation of transmission coefficient for different values of problem parameters are displayed graphically.

Key words: Sound transmission, absorbent lining, muffler, extended inlet/outlet

1. Introduction

Expansion chambers are one of the ways to reduce the unwanted noise propagating along a duct which can be a part of an exhaust or a ventilating system. Simple expansion chambers has been widely investigated in literature [1], [2], [3], [4]. In further investigations it has been shown that expansion chambers with extended inlet and outlet exhibit a desirable acoustic attenuation performance [5], [6], [7]. In [7], Selamet et al. concluded that it is possible to obtain excellent acoustic attenuation by choosing the length of extended ducts to match the resonances with the zero-attenuation frequencies of expansion chambers.

Another way to reduce the unwanted noise is to treat duct walls with acoustically absorbent linings which has been shown to be very effective in [8]. Rawlins found that the rate of acoustic attenuation depend on the specific impedance of the absorbent lining. Combining the two effective approaches, expansion chamber and absorbent lining, in a silencer consist of an expansion chamber whose walls are treated by acoustic liners has been analysed in [9].

Recently, sound transmission in a duct with an area expansion and extended inlet is investigated in [10], [11]. It has been shown that extension length and acoustic lining were both effective to control the transmitted field in the duct.

In this paper, the transmission of sound in an expansion chamber with extended inlet and outlet, where the lateral wall of expansion chamber and the outer walls of extended inlet/outlet are treated by different locally reacting lining, is investigated. The main objective of this paper is to reveal the influence of the absorbent lining on the transmitted field propagating in an expansion chamber with extended inlet/outlet. The method adopted in this paper is similar to one in [11] and consists of expanding the fields in the overlap regions into a series of eigenfunctions and using

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the Fourier transform technique elsewhere. The problem is then reduced directly into a Wiener-Hopf equation whose solution involve a set of infinitely many unknown expansion coefficients satisfying a system of linear algebraic equations. Numerical solution to these systems are obtained for various values of the parameters of the problem such as extended inlet/outlet lengths and the specific impedance of the linings whereby the effects of these parameters on the transmitted field are presented graphically.
The time dependence is assumed to be $e^{-i\omega t}$ with ω being the angular frequency and suppressed throughout this paper.

2. Formulation

Consider an infinite circular cylindrical duct splitted into two semi-infinite parts and a finite expansion chamber of different radii with common longitudinal axis, say *z*, in a cylindrical polar coordinate system (ρ, φ, z). They occupy the regions $\rho = a; z < 0 \cup \rho = a; z > l$ and $\rho = b > a$; $-l_1 < z < l_2$; respectively. $l > 0$ represents the gap length between semi-infinite parts of the central duct, while $l_1 > 0, l_2 > 0$ are extended inlet/outlet lengths respectively. Infinite duct and expansion chamber are connected with vertical walls at $z = -l_1$ and $z = l_2$. Outer parts of the extended inlet/outlet surfaces $\rho = a + 0\,; -l_1 < z < 0$ and $\rho = a + 0\,; 0 < z < l_2$ and the lateral inner surface of the expansion chamber $\rho = b - 0\,; -l_1 < z < l_2$ are assumed to be treated by acoustically absorbing linings which are characterized by constant but different surface impedances respectively, while the remaining surfaces are perfectly rigid (see Fig. 1). The waveguides are immersed in the inviscid and compressible stationary fluid of density ρ_0 and sound speed *c*. A plane sound wave is incident from the positive *z*-direction, through the waveguide of radius $\rho = a$. From the symmetry of the geometry of the problem and the incident field the scattering field everywhere will be independent of the φ coordinate. We shall therefore introduce a scalar potential $u(\rho, z)$ which defines the acoustic pressure and velocity by $p = i\omega\rho_0 u$ and **v** = grad *u*, respectively.

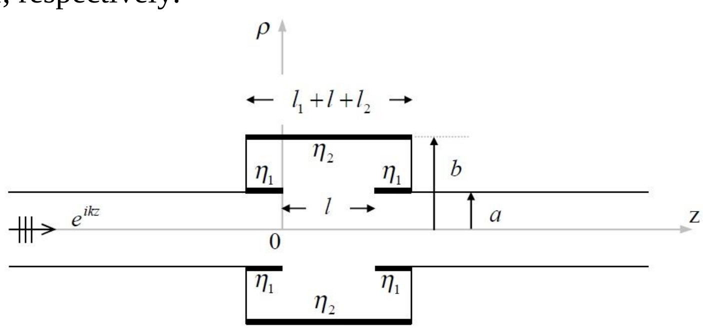

Figure 1. Geometry of the problem

It is convenient to write the total field in different regions as:

$$u^T(r,z) = \begin{cases} u_1(\rho,z) + u^i(\rho,z) & , \quad \rho < a, z \in (-\infty, \infty) \\ u_2(\rho,z) & , \quad a < \rho < b, z \in (0, l) \\ u_3(\rho,z) & , \quad a < \rho < b, z \in (-l_1, 0) \\ u_4(\rho,z) & , \quad a < \rho < b, z \in (l, l_2) \end{cases} \tag{1}$$

where u^i is the incident field and defined as,

$$u^i(\rho, z) = e^{ikz} \tag{2}$$

with $k = \omega/c$ being the wave number. For the sake of analytical convenience we will assume that the surrounding medium is slightly lossy and k has a small positive imaginary part. The lossless case can be obtained by letting $\mathrm{Im}(k) \to 0$ at the end of the analysis.

The unknown scalar potentials $u_j(\rho, z)$ $(j = 1,2,3,4)$ satisfy the Helmholtz equation in their respective regions for z.

$$\left[\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial}{\partial\rho}\right) + \frac{\partial^2}{\partial z^2} + k^2\right] u_{1,2,3,4}(\rho, z) = 0 \tag{3}$$

together with the boundary conditions and continuity equations:

$$\frac{\partial}{\partial\rho} u_1(a, z) = 0 \quad , \quad z < 0 \cup z > l \tag{4}$$

$$\left[ik\eta_2 - \frac{\partial}{\partial\rho}\right] u_2(b, z) = 0 \quad , \quad 0 < z < l \tag{5}$$

$$\frac{\partial}{\partial z} u_3(\rho, -l_2) = 0 \quad , \quad a < \rho < b \tag{6}$$

$$\left[ik\eta_1 + \frac{\partial}{\partial\rho}\right] u_3(a, z) = 0 \quad , \quad -l_1 < z < 0 \tag{7}$$

$$\left[ik\eta_2 - \frac{\partial}{\partial\rho}\right] u_3(b, z) = 0 \quad , \quad -l_1 < z < 0 \tag{8}$$

$$\left[ik\eta_1 + \frac{\partial}{\partial\rho}\right] u_4(a, z) = 0 \quad , \quad l < z < l_2 \tag{9}$$

$$\left[ik\eta_2 - \frac{\partial}{\partial\rho}\right] u_4(b, z) = 0 \quad , \quad l < z < l_2 \tag{10}$$

$$\frac{\partial}{\partial z} u_2(\rho, 0) - \frac{\partial}{\partial z} u_3(\rho, 0) = 0 \quad , \quad a < \rho < b \tag{11}$$

$$u_2(\rho, 0) - u_3(\rho, 0) = 0 \quad , \quad a < \rho < b \tag{12}$$

$$\frac{\partial}{\partial z} u_2(\rho, l) - \frac{\partial}{\partial z} u_4(\rho, l) = 0 \quad , \quad a < \rho < b \tag{13}$$

$$u_2(\rho, l) - u_4(\rho, l) = 0 \quad , \quad a < \rho < b \tag{14}$$

$$\frac{\partial}{\partial\rho}\left[u_1(a,z)+u^i(a,z)\right]-\frac{\partial}{\partial\rho}u_2(a,z)=0\ ,\qquad 0<z<l \tag{15}$$

$$u_1(a,z)+u^i(a,z)-u_2(a,z)=0\ ,\qquad 0<z<l \tag{16}$$

The above mixed boundary value problem will be solved mainly by using Fourier transform technique together with series expansion of unknown fields in extended inlet/outlet regions.

2.1. Fourier transformation

Consider the Fourier transform of the Helmholtz equation in the region $\rho < a$ for $z \in (-\infty,\infty)$, namely,

$$\left[\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial}{\partial\rho}\right)+(k^2-\alpha^2)\right]F(\rho,\alpha)=0 \tag{17}$$

where $F(\rho,\alpha)$ is the Fourier transform of the field $u_1(\rho,z)$ defined to be

$$F(\rho,\alpha)=\int_{-\infty}^{\infty}u_1(\rho,z)e^{i\alpha z}dz=F_-(\rho,\alpha)+F_1(\rho,\alpha)+e^{i\alpha l}F_+(\rho,\alpha) \tag{18}$$

While $F_+(\rho,\alpha)$ are half-plane analytical functions $F_1(\rho,\alpha)$ is an entire function on complex α-plane defined by Fourier integrals as:

$$F_+(\rho,\alpha)=\int_{l}^{\infty}u_1(r,z)e^{i\alpha(z-l)}dz\,,\qquad F_-(\rho,\alpha)=\int_{-\infty}^{0}u_1(r,z)e^{i\alpha z}dz\,,\qquad F_1(\rho,\alpha)=\int_{0}^{l}u_1(r,z)e^{i\alpha z}dz \tag{19}$$

Owing to the analytical properties of $F_\pm(\rho,\alpha)$ and $F_1(\rho,\alpha)$ the solution of (17) reads

$$F_-(\rho,\alpha)+F_1(\rho,\alpha)+e^{i\alpha l}F_+(\rho,\alpha)=-\dot{F}_1(a,\alpha)\frac{J_0(K\rho)}{K(\alpha)J_1(Ka)} \tag{20}$$

where $K(\alpha)=\sqrt{k^2-\alpha^2}$ is the square root function defined by $K(0)=k$. The dot $(\cdot)$ over F represents the derivative with respect to ρ and J_n stands for the Bessel function of integer order. In the region $a < \rho < b$ for $z \in (0, l)$, finite Fourier transform of the Helmholtz equation becomes

$$\left[\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial}{\partial\rho}\right)+K^2(\alpha)\right]G_1(\rho,\alpha)$$
$$=\left[\frac{\partial}{\partial z}u_2(\rho,0)-i\alpha u_2(\rho,0)\right]-e^{i\alpha l}\left[\frac{\partial}{\partial z}u_2(\rho,l)-i\alpha u_2(\rho,l)\right] \tag{21}$$

where $G_1(\rho,\alpha)$ is an entire function on the α-plane which is defined as

$$G_1(\rho,\alpha)=\int_{0}^{l}u_2(\rho,z)e^{i\alpha z}dz \tag{22}$$

The solution of (21) in terms of $f(t)=\frac{\partial}{\partial z}u_2(t,0), g(t)=u_2(t,0), p(t)=\frac{\partial}{\partial z}u_2(t,l), q(t)=u_2(t,l)$

$$G_1(\rho,\alpha)=-\frac{1}{K(\alpha)L(\alpha)}\Bigg\{\dot{F}_1(a,\alpha)[J_0(K\rho)Y(b,\alpha)-Y_0(K\rho)J(b,\alpha)]$$
$$-\int_{a}^{b}\{[f(t)-i\alpha g(t)]-e^{i\alpha l}[p(t)-i\alpha q(t)]\}\mathcal{H}(t,\rho,\alpha)\,t\,dt\Bigg\} \tag{23}$$

where $\mathcal{H}(t,\rho,\alpha)$ is the corresponding Green's function found to be,

$$\mathcal{H}(t,\rho,\alpha) = \frac{\pi}{2}K(\alpha)\begin{cases}[J_0(Kt)Y_1(Ka)-Y_0(Kt)J_1(Ka)][J_0(K\rho)Y(b,\alpha)-Y_0(K\rho)J(b,\alpha)] & , \quad a\le t\le\rho \\ [J_0(K\rho)Y_1(Ka)-Y_0(K\rho)J_1(Ka)][J_0(Kt)Y(b,\alpha)-Y_0(Kt)J(b,\alpha)] & , \quad \rho\le t\le b\end{cases} \tag{24}$$

and $J(b,\alpha), Y(b,\alpha), L(\alpha)$ are defined as

$$L(\alpha) = [J_1(Ka)Y(b,\alpha) - Y_1(Ka)J(b,\alpha)] \tag{25}$$

$$J(b,\alpha) = ik\eta_2 J_0(Kb) + KJ_1(Kb) \tag{26}$$

$$Y(b,\alpha) = ik\eta_2 Y_0(Kb) + KY_1(Kb) \tag{27}$$

Since the left side of (23) is an entire function so do the right side has to be an entire function too. This can be carried out by equating the residual contribution at the zeros of $K(\alpha)L(\alpha)$ to zero by the following relation

$$\dot{F}_1(a,\pm\alpha_m) = \frac{\pi}{2}\frac{K_m J_1(K_m a)}{J(b,\alpha)}\left\{\int_a^b \{[f(t)\mp i\alpha_m g(t)] - e^{\pm i\alpha_m l}[p(t)\mp i\alpha_m q(t)]\} \times [J_0(Kt)Y(b,\alpha)-Y_0(Kt)J(b,\alpha)]\,tdt\right\} \quad m=1,2,.. \tag{28}$$

where $K(\pm\alpha_m) = K_m$ $(m = 1,2 \ldots)$ are the zeros of the function $K(\alpha)L(\alpha)$.

Now using continuity relations (15) and (16) we obtain the following equation valid in the strip $Im(-k) < Im(\alpha) < Im(k)$,

$$\frac{2}{a}\dot{F}_1(a,\alpha)\frac{V(\alpha)}{K^2(\alpha)} + F_-(a,\alpha) + e^{i\alpha l}F_+(a,\alpha) = \frac{1}{aK(\alpha)L(\alpha)}\int_a^b \{[f(t)-i\alpha g(t)] - e^{i\alpha l}[p(t)-i\alpha q(t)]\}[J_0(Kt)Y(b,\alpha)-Y_0(Kt)J(b,\alpha)]\,tdt - \frac{1-e^{i(k+\alpha)l}}{i(k+\alpha)} \tag{29}$$

Here, $V(\alpha)$ stands for the kernel function defined and factorized as

$$V(\alpha) = \frac{J(b,\alpha)}{\pi J_1(Ka)[J_1(Ka)Y(b,\alpha)-Y_1(Ka)J(b,\alpha)]} = V_+(\alpha)V_-(\alpha) \tag{30}$$

where $V_+(\alpha)$ and $V_-(\alpha)$ stand for the split functions regular and free of zeros in the upper and lower half-planes of the complex α-plane, respectively. Their explicit expressions can be found in [12].

2.2. Series Expansion and Wiener-Hopf Equation

The unknown field $u_3(\rho,z)$ and $u_4(\rho,z)$ can be expressed is terms of the waveguide modes as

$$u_3(\rho,z) = \sum_{n=1}^{\infty} a_n cos[\beta_n(z+l_1)][J_0(\gamma_n\rho) - R_n Y_0(\gamma_n\rho)] \tag{31}$$

$$u_4(\rho,z) = \sum_{n=1}^{\infty} b_n cos[\beta_n(z-l_2)][J_0(\gamma_n\rho) - R_n Y_0(\gamma_n\rho)] \tag{32}$$

with

$$R_n = \frac{ik\eta_1 J_0(\gamma_n a) - \gamma_n J_1(\gamma_n a)}{ik\eta_1 Y_0(\gamma_n a) - \gamma_n Y_1(\gamma_n a)} = \frac{ik\eta_2 J_0(\gamma_n b) + \gamma_n J_1(\gamma_n b)}{ik\eta_2 Y_0(\gamma_n b) + \gamma_n Y_1(\gamma_n b)} \tag{33}$$

where $\eta_{1,2}$ are admittance values related with impedances as $Z_i = \frac{1}{\eta_i}$ $(i = 1,2)$ and γ_n's are the roots of the equation

$$\frac{ik\eta_1 J_0(\gamma_n a) - \gamma_n J_1(\gamma_n a)}{ik\eta_1 Y_0(\gamma_n a) - \gamma_n Y_1(\gamma_n a)} - \frac{ik\eta_2 J_0(\gamma_n b) + \gamma_n J_1(\gamma_n b)}{ik\eta_2 Y_0(\gamma_n b) + \gamma_n Y_1(\gamma_n b)} = 0 \tag{34}$$

while β_n's are defined as

$$\beta_n = \sqrt{k^2 - \gamma_n^2}\ , \quad n = 1,2,\ldots \tag{35}$$

Taking into account continuity relations (11), (12), (13) and (14) together with the expressions (31) and (32), we are able to define $f(t), g(t), p(t)$ and $q(t)$ in terms of the unknown coefficients a_n, b_n.

$$f(t) = -\sum_{n=1}^{\infty} a_n \beta_n sin(\beta_n l_1)[J_0(\gamma_n t) - R_n Y_0(\gamma_n t)] \tag{36}$$

$$g(t) = \sum_{n=1}^{\infty} a_n cos(\beta_n l_1)[J_0(\gamma_n t) - R_n Y_0(\gamma_n t)] \tag{37}$$

and

$$p(t) = -\sum_{n=1}^{\infty} b_n \beta_n sin[\beta_n(l - l_2)][J_0(\gamma_n t) - R_n Y_0(\gamma_n t)] \tag{38}$$

$$q(t) = \sum_{n=1}^{\infty} b_n cos[\beta_n(l - l_2)][J_0(\gamma_n t) - R_n Y_0(\gamma_n t)] \tag{39}$$

Substituting the series expansions (36), (37), (38) and (39) into (28) and (29) we obtain

$$\begin{aligned}\dot{F}_1(a, \pm\alpha_m) = -ik\eta_1 \Bigg\{ &\sum_{n=1}^{\infty} a_n[\beta_n sin(\beta_n l_1) \pm i\alpha_m cos(\beta_n l_1)]\Delta_{mn} \\ &- e^{\pm i\alpha_m l} \sum_{n=1}^{\infty} b_n\big[\beta_n sin[\beta_n(l - l_2)] \pm i\alpha_m cos[\beta_n(l - l_2)]\big]\Delta_{mn} \Bigg\} \quad m = 1,2,..\end{aligned} \tag{40}$$

where

$$\Delta_{mn} = \frac{[J_0(\gamma_n a) - R_n Y_0(\gamma_n a)]}{\alpha_m^2 - \beta_n^2} \tag{41}$$

and the following equation is Modified Wiener-Hopf equation to be solved by using nalytical properties of (+) and (-) functions,

$$\begin{aligned}\frac{2}{a}\dot{F}_1(a,\alpha)\frac{V(\alpha)}{K^2(\alpha)} + F_-(a,\alpha) + e^{i\alpha l}F_+(a,\alpha) = \frac{W(\alpha)}{K(\alpha)L(\alpha)} &\sum_{n=1}^{\infty} \{a_n[\beta_n sin(\beta_n l_1) + i\alpha cos(\beta_n l_1)] \\ - e^{i\alpha l} b_n\big[\beta_n sin[\beta_n(l - l_2)] + i\alpha cos[\beta_n(l - l_2)]\big]\} &\frac{[J_0(\gamma_n a) - R_n Y_0(\gamma_n a)]}{\alpha^2 - \beta_n^2} - \frac{1 - e^{i(k+\alpha)l}}{i(k+\alpha)}\end{aligned} \tag{42}$$

where

$$W(\alpha) = [J(a,\alpha)Y(b,\alpha) - Y(a,\alpha)J(b,\alpha)] \tag{43}$$

$$J(a,\alpha) = ik\eta_1 J_0(Ka) - KJ_1(Ka) \tag{44}$$

$$Y(a,\alpha) = ik\eta_1 Y_0(Ka) - KY_1(Ka) \tag{45}$$

2.3. Solution of Wiener-Hopf Equation and Determination of Unknown Coefficients

Following a similar procedure defined in [9], we first write (42) in the following form

$$\frac{2}{a}\dot{F}_1(a,\alpha)\frac{V(\alpha)}{K^2(\alpha)}+e^{i\alpha l}M(\alpha)+N(\alpha)=0 \tag{46}$$

$M(\alpha)$ and $N(\alpha)$ are defined respectively as,

$$M(\alpha)=F_+(a,\alpha)+\frac{W(\alpha)}{K(\alpha)L(\alpha)}\sum_{n=1}^{\infty}b_n\big[\beta_n sin[\beta_n(l-l_2)]+i\alpha cos[\beta_n(l-l_2)]\big]\frac{[J_0(\gamma_n a)-R_nY_0(\gamma_n a)]}{\alpha^2-\beta_n^2}-\frac{e^{ikl}}{i(k+\alpha)} \tag{47}$$

$$N(\alpha)=F_-(a,\alpha)-\frac{W(\alpha)}{K(\alpha)L(\alpha)}\sum_{n=1}^{\infty}a_n[\beta_n sin(\beta_n l_1)+i\alpha cos(\beta_n l_1)]\frac{[J_0(\gamma_n a)-R_nY_0(\gamma_n a)]}{\alpha^2-\beta_n^2}+\frac{1}{i(k+\alpha)} \tag{48}$$

and then applying factorization and decomposition procedures we arrive at the solution for $M(\alpha)$ and $N(\alpha)$.

$$\frac{(k+\alpha)}{V_+(\alpha)}M(\alpha)=\pi\sum_{n=1}^{\infty}\frac{J_1^2(\delta_n a/b)Y(b,\gamma_n)(k-\gamma_n)V_+(\gamma_n)e^{i\gamma_n l}}{J'(b,\gamma_n)(\gamma_n+\alpha)}N(-\gamma_n)+\frac{2}{\pi a}\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{J(b,\alpha_m)}{K_mJ_1(K_ma)}\frac{ik\eta_1(k+\alpha_m)}{V_+(\alpha_m)(\alpha_m-\alpha)}\frac{\big[\beta_n sin[\beta_n(l-l_2)]+i\alpha_m cos[\beta_n(l-l_2)]\big]\Delta_{mn}}{[KJ_1(Ka)Y(b,\alpha)-KY_1(Ka)J(b,\alpha)]'_{\alpha=\alpha_m}}b_n \tag{49}$$

$$\frac{(k-\alpha)}{V_-(\alpha)}N(\alpha)=\pi\sum_{n=1}^{\infty}\frac{J_1^2(\delta_n a/b)Y(b,\gamma_n)(k-\gamma_n)V_+(\gamma_n)e^{i\gamma_n l}}{J'(b,\gamma_n)(\gamma_n-\alpha)}M(\gamma_n)-\frac{2k}{iV_+(k)(k+\alpha)}-\frac{2}{\pi a}\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{J(b,\alpha_m)}{K_mJ_1(K_ma)}\frac{ik\eta_1(k+\alpha_m)}{V_+(\alpha_m)(\alpha_m+\alpha)}\frac{[\beta_n sin(\beta_n l_1)-i\alpha_m cos(\beta_n l_1)]\Delta_{mn}}{[KJ_1(Ka)Y(b,\alpha)-KY_1(Ka)J(b,\alpha)]'_{\alpha=\alpha_m}}a_n \tag{50}$$

From (46), (49) and (50) the solution of W-H equation is found to be

$$\begin{aligned}\frac{2}{a}\dot{F}_1(a,\alpha)=&-\pi\frac{(k+\alpha)}{V_+(\alpha)}\sum_{n=1}^{\infty}\frac{J_1^2(\delta_n a/b)Y(b,\gamma_n)(k-\gamma_n)V_+(\gamma_n)e^{i\gamma_n l}}{J'(b,\gamma_n)(\gamma_n-\alpha)}M(\gamma_n)\\&+\frac{2}{\pi a}\frac{(k+\alpha)}{V_+(\alpha)}\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{J(b,\alpha_m)}{K_mJ_1(K_ma)}\frac{ik\eta_1(k+\alpha_m)}{V_+(\alpha_m)(\alpha_m+\alpha)}\frac{[\beta_n sin(\beta_n l_1)-i\alpha_m cos(\beta_n l_1)]\Delta_{mn}}{[KJ_1(Ka)Y(b,\alpha)-KY_1(Ka)J(b,\alpha)]'_{\alpha=\alpha_m}}a_n-\\&\frac{2}{\pi a}e^{i\alpha l}\frac{(k-\alpha)}{V_-(\alpha)}\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{J(b,\alpha_m)}{K_mJ_1(K_ma)}\frac{ik\eta_1(k+\alpha_m)}{V_+(\alpha_m)(\alpha_m-\alpha)}\frac{\big[\beta_n sin[\beta_n(l-l_2)]+i\alpha_m cos[\beta_n(l-l_2)]\big]\Delta_{mn}}{[KJ_1(Ka)Y(b,\alpha)-KY_1(Ka)J(b,\alpha)]'_{\alpha=\alpha_m}}b_n\\&-\pi e^{i\alpha l}\frac{(k-\alpha)}{V_-(\alpha)}\sum_{n=1}^{\infty}\frac{J_1^2(\delta_n a/b)Y(b,\gamma_n)(k-\gamma_n)V_+(\gamma_n)e^{i\gamma_n l}}{J'(b,\gamma_n)(\gamma_n+\alpha)}N(-\gamma_n)-\frac{2k}{iV_+(k)V_+(\alpha)}\end{aligned} \tag{51}$$

The solution involves unknown coefficients $a_n, b_n, M(\gamma_n), N(-\gamma_n)$. To determine these coefficents we construct a system of linear algebraic equations by substituting $\alpha=\pm\alpha_r$ in (51),

$\alpha=\gamma_r$ in (49) and $\alpha=-\gamma_r$ in (50) respectively. Using the relation in (40) we achieve the following infinite system of linear equations

$$ik\eta_1\sum_{n=1}^{\infty}[\beta_n sin(\beta_n l_1)+i\alpha_r cos(\beta_n l_1)]\Delta_{rn}a_n$$
$$=-\frac{\pi a}{2}\frac{(k+\alpha_r)}{V_+(\alpha_r)}\sum_{n=1}^{\infty}\frac{J_1^2(\delta_n a/b)Y(b,\gamma_n)(k-\gamma_n)V_+(\gamma_n)e^{i\gamma_n l}}{J'(b,\gamma_n)(\gamma_n-\alpha_r)}M(\gamma_n)$$
$$+\frac{1}{\pi}\frac{(k+\alpha_r)}{V_+(\alpha_r)}\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{J(b,\alpha_m)}{K_mJ_1(K_ma)}\frac{ik\eta_1(k+\alpha_m)}{V_+(\alpha_m)(\alpha_m+\alpha_r)}\frac{[\beta_n sin(\beta_n l_1)-i\alpha_m cos(\beta_n l_1)]\Delta_{mn}}{[KJ_1(Ka)Y(b,\alpha)-KY_1(Ka)J(b,\alpha)]'_{\alpha=\alpha_m}}a_n$$
$$+\frac{ka}{iV_+(k)V_+(\alpha_r)} \qquad (52)$$

$$-ik\eta_1\sum_{n=1}^{\infty}\big[\beta_n sin[\beta_n(l-l_2)]-i\alpha_r cos[\beta_n(l-l_2)]\big]\Delta_{rn}b_n$$
$$=\frac{\pi a}{2}\frac{(k+\alpha_r)}{V_+(\alpha_r)}\sum_{n=1}^{\infty}\frac{J_1^2(\delta_n a/b)Y(b,\gamma_n)(k-\gamma_n)V_+(\gamma_n)e^{i\gamma_n l}}{J'(b,\gamma_n)(\gamma_n-\alpha_r)}N(-\gamma_n)+$$
$$\frac{1}{\pi}\frac{(k+\alpha_r)}{V_+(\alpha_r)}\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{J(b,\alpha_m)}{K_mJ_1(K_ma)}\frac{ik\eta_1(k+\alpha_m)}{V_+(\alpha_m)(\alpha_m+\alpha_r)}\frac{\big[\beta_n sin[\beta_n(l-l_2)]+i\alpha_m cos[\beta_n(l-l_2)]\big]\Delta_{mn}}{[KJ_1(Ka)Y(b,\alpha)-KY_1(Ka)J(b,\alpha)]'_{\alpha=\alpha_m}}b_n \qquad (53)$$

$$\frac{(k+\gamma_r)}{V_+(\gamma_r)}M(\gamma_r)=\pi\sum_{n=1}^{\infty}\frac{J_1^2(\delta_n a/b)Y(b,\gamma_n)(k-\gamma_n)V_+(\gamma_n)e^{i\gamma_n l}}{J'(b,\gamma_n)(\gamma_n+\gamma_r)}N(-\gamma_n)$$
$$+\frac{2}{\pi a}\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{J(b,\alpha_m)}{K_mJ_1(K_ma)}\frac{ik\eta_1(k+\alpha_m)}{V_+(\alpha_m)(\alpha_m-\gamma_r)}\frac{\big[\beta_n sin[\beta_n(l-l_2)]+i\alpha_m cos[\beta_n(l-l_2)]\big]\Delta_{mn}}{[KJ_1(Ka)Y(b,\alpha)-KY_1(Ka)J(b,\alpha)]'_{\alpha=\alpha_m}}b_n \qquad (54)$$

$$\frac{(k+\gamma_r)}{V_+(\gamma_r)}N(-\gamma_r)=\pi\sum_{n=1}^{\infty}\frac{J_1^2(\delta_n a/b)Y(b,\gamma_n)(k-\gamma_n)V_+(\gamma_n)e^{i\gamma_n l}}{J'(b,\gamma_n)(\gamma_n+\gamma_r)}M(\gamma_n)-\frac{2k}{iV_+(k)(k-\gamma_r)}$$
$$-\frac{2}{\pi a}\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{J(b,\alpha_m)}{K_mJ_1(K_ma)}\frac{ik\eta_1(k+\alpha_m)}{V_+(\alpha_m)(\alpha_m-\gamma_r)}\frac{[\beta_n sin(\beta_n l_1)-i\alpha_m cos(\beta_n l_1)]\Delta_{mn}}{[KJ_1(Ka)Y(b,\alpha)-KY_1(Ka)J(b,\alpha)]'_{\alpha=\alpha_m}}a_n \qquad (55)$$

We solve linear algebraic system in (52), (53), (54) and (55) numerically by truncating the infinite series at some number N.

2.4. Reflection and Transmission Coefficients

The scattered field $u_1(\rho,z)$ can be obtained by taking the inverse Fourier transform of $F(\rho,\alpha)$. From the definition (18) and solution (20) we can write,

$$u_1(\rho,z)=-\frac{1}{2\pi}\int_{-\infty}^{\infty}\dot{F}_1(a,\alpha)\frac{J_0(K\rho)}{K(\alpha)J_1(Ka)}e^{-i\alpha z}d\alpha \qquad (56)$$

The evaluation of this integral for $z<0$ and $z>l$ will give us the reflected wave and the transmitted wave, respectively. The reflection coefficient R of the fundamental mode is defined as the complex coefficient of the term exp(-i*kz*) and is computed from the contribution of the first pole at $\alpha = k$. The result is

$$R = \frac{i}{ka} \dot{F}_1(a, k) \tag{57}$$

Similarly, the transmission coefficient T of the fundamental mode which is defined as to be the complex coefficient of exp(i*kz*) is calculated from the contribution at the pole $\alpha = -k$ as,

$$T = \frac{i}{ka} \dot{F}_1(a, -k) \tag{58}$$

3. Results and Discussion

In order to show the effects of the parameters as extended inlet/outlet lengths l_1, l_2 and the surface admittances $\eta_{1,2}$ on the sound transmission, some numerical results showing the variation of transmission coefficient T are presented. In all numerical calculations the solution of the infinite system of algebraic equations is obtained by truncating the infinite series at some number N. Only imaginary values of surface admittances s.t. $\eta_{1,2} = iX_{1,2}$, $X \in \mathbb{R}$ were taken. In all graphs Transmission Loss is found by the following definition: $TL = -20 \log_{10}|T|$.

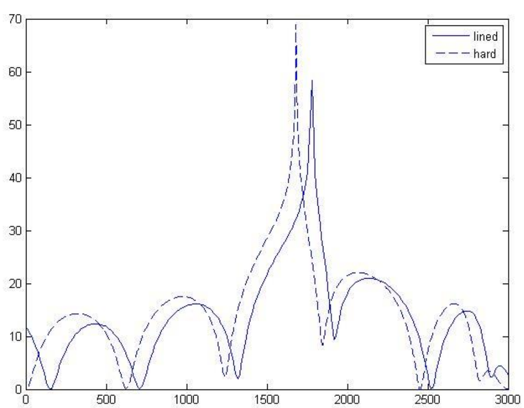

Figure 2. Transmission Loss(dB) versus frequency(Hz) for $\eta_1 = \eta_2 = i0.1$, $l_1 = 0$, $l_2 = 4.0$, $l = 28.23$ (cm)

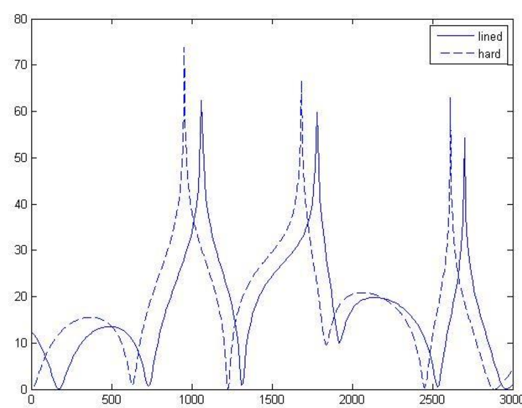

Figure 3. Transmission Loss(dB) versus frequency(Hz) for $\eta_1 = \eta_2 = i0.1$, $l_1 = 8.0$, $l_2 = 4.0$, $l = 28.23$ (cm)

In Fig. 2, Fig. 3 and Fig. 4 it is observed that the lining admittances affect the resonance peaks. Number of domes does not change, but some attenuation in the peaks are seen for lined case. Hard cases in all figures are in agreement with Figure 3, Figure 4 and Figure 6 of [7].

Conclusions

This paper examines the transmission of sound waves in an expansion chamber with lined extended inlet/outlet and lined lateral walls. Problem is solved by a hybrid method of formulation consisting of expressing the fields in extended inlet/outlet regions in terms of waveguide modes and using the Fourier transform elsewhere is adopted. The mixed boundary value problem is reduced to a Modified Wiener-Hopf equation whose solution involves infinitely many expansion coefficients satisfying a system of linear algebraic equations. These equations are solved numerically and the effects of problem parameters on transmitted field are displayed graphically.

Acknowledgements

The author would like to thank Karabuk University Scientific Research Projects (BAP) Coordinatorship for their support to participate in ISITES2017.

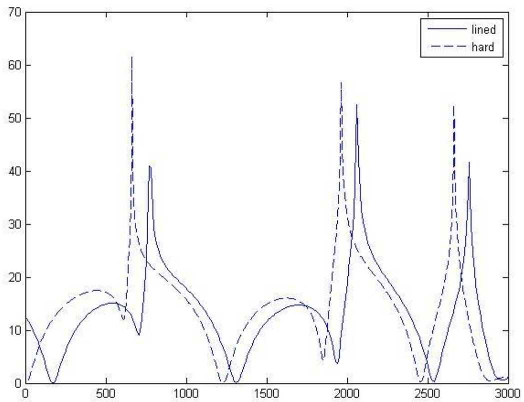

Figure 4. Transmission Loss(dB) versus frequency(Hz) for $\eta_1 = \eta_2 = i0.1, l_1 = 12.0$, $l_2 = 0$, $l = 28.23$ (cm).

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